What is Basic Fact Fluency?

The National Research Council (2001) described computational or fact fluency as "the efficient, appropriate, and flexible application of single-digit and multidigit calculation skills- is an essential aspect of mathematical proficiency". According to Baroody (2006), there are three phases children typically progress through in mastering basic number combinations. The phases are:

Phase 1: *Counting strategies*- using object counting (e.g., with blocks, fingers, marks) or verbal counting to determine an answer. For example, children might solve 5 + 7 by starting from 5 and counting on 7 more to get 12.
Phase 2: *Reasoning strategies*- using known information (e.g., known facts and relationships) to logically determine the answer of an unknown combination. Children might solve 5 + 7 by starting from the easier, known fact 5 + 5 = 10 and then just adding 2 more to get 12.
Phase 3: *Mastery*- efficient (fast and accurate) production of answers. Children respond "12" with little/no hesitation.

Similar to Baroody's phases, Haring and Eaton (1978) proposed stages of learning that appear to parallel the phases. The stages are: Acquisition, Building Fluency, and Generalization and Adaptation. In the Acquisition stage, student responses are typically slow with frequent errors. Before allowing independent practice, it is recommended that students demonstrate accuracy of 90 to 95 percent. Otherwise, frustration results and students do not benefit from the independent practice. In the Generalization and Adaptation stage, students demonstrate mastery and begin to apply previously learned skills or strategies to solve novel problems.

What is Procedural Fluency?

Previously, procedural fluency was defined several ways usually dependent on the author's perspective of the term. According to definitions provided by the National Council of Teachers of Mathematics and the Common Core State Standards (2010), procedural fluency is the ability to: apply the appropriate strategy when solving problems, demonstrate flexibility in thinking, and complete problems efficiently and accurately. The four elements of **flexibility, correct strategy selection, efficiency, and accuracy** are critical for procedural fluency.

Why is Basic Fact Fluency Important?

Baroody (2006) stated most educators agree that children should master basic number combinations (phase 3) but there is disagreement how to achieve mastery or how the combinations are learned. According to Kling (2011), the goal of fact fluency is to find efficient, effective ways to apply *known* facts to *derive* unknown facts. Use of basic facts leads to development of procedural fluency to solve more complex problems. Hattie and Yates (2013) suggest that when basic skills are lacking, **cognitive load** or mental effort is greater when problems require dedicated effort to solve. Knowledge of facts can allow for a shift of cognitive energy and allow individuals to more efficiently solve complex problems.

What are the research based methods of math fact instruction and assessment?
Bay-Williams and Kling (2016) refer to two methods of fact instruction and assessment. The first method is Memorize, Test, Continue (M-T-C). In this method, children memorize a series of facts, are tested, then move to the next set of facts. Research suggests this method of instruction and assessment may have negative impacts on children's mathematical confidence and view of mathematics (Boaler, 2014; Ramirez et al., 2013). Furthermore, this model only provides teachers with information if students are accurately solving problems in a rapid manner. Aligning to Haring and Eaton's stages of learning, this model may not allow for adequate skill acquisition before moving to fluency possibly precluding generalization.

The other method of instruction and assessment described by Bay-Williams and Kling is Reasoning Strategies, Practice, and Monitoring (R-P-M). In this model, students learn reasoning strategies and apply the strategies in games or other situations while the teacher monitors how children are progressing through groups of facts. This model allows teachers to assess whether students are engaging in flexible thinking, selecting the correct strategy, efficiently solving problems and accurately solving problems.

The critical component addressed in the R-P-M model is the emphasis on thinking strategies to solve problems. Isaacs and Carroll (1999) challenged rote fact memorization and suggested strategy based instruction of math facts as a more effective approach. In general, Isaacs and Carroll proposed providing problems and allowing students the opportunity and guidance to solve problems. Through meaningful repetition, fact fluency would be mastered. Since that time, games and other activities have been supported through research as a preferred method for students to achieve math fact fluency, particularly when purposeful and frequently used, with a focus on strategy application and discussion (Boaler, 2015;Kling, 2011; Bay-Williams & Kling, 2015; Kling & Bay-Williams, 2014). Previous studies support that students in R-P-M classrooms outperform their peers in M-T-C classes (Thornton, 1978; Kling, 2011). Moreover, findings from Dunlosky et al, (2013) suggest that untimed, low- or no- stakes practice tests have been found to improve student achievement, particularly when feedback is provided.

A recent study by Brendefur et al. (2015) was conducted by teaching students multiplication facts using flexible representations. The premise was that when students engage in flexible and conceptual approaches to fact mastery, their fluency increases (Gray, Pitta, & Tall, 2000; Mulligan & Mitchelmore, 2009). Results from Brendefur et al. revealed that students in grades 3, 4, and 5 with 10-15 minutes of strategy based fact fluency practice a day outperformed those in drill based fluency practice.

Recalling Baroody's phases of fact learning, the M-T-C model risks moving too quickly from phase 1 (counting) to phase 3 (mastery) without students developing strong knowledge of strategies. Without effective practice, students may lack a conceptual understanding how to use facts to derive answers for unknown combinations. In the R-P-M model, teachers need to provide structured activities to reinforce mastery of fact groups while assessing and monitoring progress.

What are the recommended methods to assess basic fact fluency?

Although less standardized and objective in appearance, recent research supports use of multiple informal sources to assess math fact mastery (Kling & Bay-Williams, 2014; Bay-Williams & Kling, 2015):

1. Student Interviews- students explain how they solved problem
2. Observations- recognize strategy used
3. Journaling- express flexibility of problem solving
4. Quizzes- assure that students know facts before moving on

If timed tests are continued, strategies to consider:

1. Incorporate more frequent, low or no stakes tests that students score individually but do not turn in
2. Teacher feedback provided
3. Peer assisted classroom activities focused on math fact mastery

References (click to access source document, when available):

Bay-Williams, J. & Kling, G. (2015). Developing Fact Fluency. Turn Off Timers, Turn Up Formative Assessment. In NCTM *Annual Perspectives in Mathematics Education (APME) 2015: Assessment to enhance learning and teaching.* Chris Suurtamm, (Ed.) National Council of Teachers of Mathematics, Reston, VA.

Baroody, Arthur J. (2006). Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them. *Teaching Children Mathematics* 13 (1). 22–31.

Boaler, J.(2014). Research Suggests That Timed Tests Cause Math Anxiety. *Teaching Children Mathematics.* 20 (8). 469–74.

Boaler, J. (2015). *What's Math Got To Do With It? How Teachers and Parents Can Transform Mathematics Learning and Inspire Success*. New York: Penguin.

Brendefur, J., Strother, S., Thiede, K., & Appleton, S. (2015). Developing multiplication fact fluency. *Advances in Social Sciences Research Journal*. 2 (8). 142-154.

Gray, E., Pitta, D., & Tall, D. (2000). Objects, actions, and images: A perspective on early number development. The Journal of Mathematical Behavior, 18(4), 401-413.

Haring, N. G., & Eaton, M. D. (1978). *Systematic instructional procedures: An instructional hierarchy. The fourth R: Research in the classroom.* 23-40. (link to Elmbrook MTSS handout)

Hattie, J., & Yates, G. (2013). *Visible Learning And The Science Of How We Learn*. United Kingdom: Routledge.

Kling, G. (2011). Fluency with basic addition. *Teaching Children Mathematics*. 18 (2). 80-88.

Kling, G. & Bay-Williams, J. M. (2014). Assessing basic fact fluency. Teaching Children Mathematics. 20 (8). 489-497.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, 21(2), 33-49.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, DC: Authors.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J.Kilpatrick, J. Swafford, and B.Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. C. (2013) Math Anxiety, Working Memory, and Math Achievement in Early Elementary School. *Journal of Cognition and Development* 14 (2). 187–202.

Thornton, C. A. (1978). Emphasizing thinking strategies in basic fact instruction. *Journal for Research in Mathematics Education*. 9, 214-227